Algebra 2

4-01 Add, Subtract, and Multiply Polynomials (4.2)

Adding and subtracting polynomials

- Add or subtract the _____ of the terms with the _____ power.
- Called _____ like terms.

Simplify $(5x^2 + x - 7) + (-3x^2 - 6x - 1)$

$(3x^3 + 8x^2 - x - 5) - (5x^3 - x^2 + 17)$

Multiplying polynomials

- Use the _____ property

Simplify $(x - 3)(x + 4)$

$(x + 2)(x^2 + 3x - 4)$

$(x - 1)(x + 2)(x + 3)$

Special Product Patterns

Sum and Difference: $(a - b)(a + b) =$ _____

Square of a Binomial: $(a \pm b)^2 =$ _____

Cube of a Binomial: $(a \pm b)^3 =$ _____

Simplify $(x + 2)^2$

$(x - 3)^2$

166 #1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 29, 33, 37, Mixed Review = 20

Algebra 2

4-02 Factor and Solve Polynomial Equations (4.4)

How to Factor

1. **Greatest Common Factor**
   - Comes from the _____ property
   - If the _____ number or variable is in each of the terms, you can bring the number to the _____ times everything that is left.

     $3x^2y + 6xy - 9xy^2$

   - Look for this _____!
2. **Check to see how many terms**
   a. *Two terms (formulas)*
      - Difference of Squares: $a^2 - b^2 = (a - b)(a + b)$

        $9x^2 - y^4$

      - Sum of Two Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

        $8x^3 + 27$

      - Difference of Two Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

        $y^3 - 8$

   b. *Three terms (General Trinomials → $ax^2 + bx + c$)*
      i. Write two sets of parentheses ( )( )
      ii. Guess and Check
      iii. The Firsts multiply to make ax^2
      iv. The Lasts multiply to make c
      v. The Outers + Inners make bx

      $x^2 + 7x + 10$ $6x^2 - 7x - 20$

   c. *Four terms (Grouping)*
      i. Group the terms into sets of two so that you can factor a common factor out of each set
      ii. Then factor the factored sets (Factor twice)

$b^3 - 3b^2 - 4b + 12$

3. **Try factoring more!**

$a^2x - b^2x + a^2y - b^2y$

$3a^2z - 27z$

$n^4 - 81$

Solving Equations by Factoring

1. Make _____
2. _____
3. Make each factor _____ because if one factor is zero, 0 time anything = 0

$2x^5 = 18x$

180 #1, 5, 9, 13, 17, 21, 25, 29, 33, 49, 53, 188 #1, 3, 5, 7, Mixed Review = 20

Algebra 2

4-03 Divide Polynomials (4.3)

Polynomial Long Division

1. Set up the division problem. _____)_____
2. _____ the _____ term of the dividend by the _____ term of the divisor.
3. _____ the answer by the divisor and write it below the like terms of the dividend.
4. _____ the bottom from the top.
5. _____ the next term of the dividend.
6. _____ steps 2–5 until reaching the last term of the dividend.
7. If the remainder is not zero, write it as a _____ using the divisor as the denominator.

$\frac{y^4+2y^2-y+5}{y^2-y+1}$

$\frac{x^3+4x^2-3x+10}{x+2}$

Synthetic Division

- _____ form of long division for dividing by a _____
- Only when dividing by _____

How to do Synthetic Division

To divide a polynomial by $x - k$,

1. Write _____ for the divisor.
2. Write the _____ of the dividend.
3. Bring the _____ coefficient down.
4. _____ the lead coefficient by k. Write the product in the next column.
5. _____ the terms of the second column.
6. _____ the result by k. Write the product in the next column.
7. _____ steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the _____. The number in the last column is the remainder, the next number from the right has degree 0, the next number from the right has degree 1, and so on. The quotient is always _____ degree less than the dividend.

Synthetic Division

$(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2)/(x + 4)$

$(y^5 + 32) \div (y + 2)$

173 #1, 3, 5, 7, 9, 11, 13, 15, 21, 31, Mixed Review = 15

Algebra 2

4-04 Find Rational Zeros of Polynomial Functions (4.5)

The Remainder Theorem

- If a polynomial $f(x)$ is _____ by $x - k$, then the remainder is the value _____.

Use the Remainder Theorem to Evaluate a Polynomial

- To evaluate polynomial $f(x)$ at $x = k$ using the Remainder Theorem,

1. Use _____ division to divide the polynomial by $x - k$.
2. The _____ is the value $f(k)$.

Use the remainder theorem to evaluate $f(x) = 3x^4 - 5x^3 + x - 14$ at $x = 2$.

The Factor Theorem

- According to the *Factor Theorem*, k is a zero of $f(x)$ if and only if $(x - k)$ is a _____ of $f(x)$.

Use the Factor Theorem to Solve a Polynomial Equation

1. Use _____ division to divide the polynomial by the given _____ , $(x - k)$.
2. _____ that the remainder is 0.
3. If the quotient is _____ a quadratic, repeat steps 1 and 2 with _____ factor using the quotient as the polynomial.
4. If the quotient _____ a quadratic, _____ the quadratic quotient if possible.
5. Set each factor equal to _____ and solve for x.

Show that $x - 2$ is a factor of $x^3 + 7x^2 + 2x - 40$. Then find the remaining factors.

Show that $x + 2$ and $x - 1$ are factors of $x^4 - 4x^3 - 3x^2 + 14x - 8$. Then find the remaining factors.

Worksheet

Algebra 2

4-05 Find All Zeros of Polynomial Functions (4.6)

Rational Zero Theorem

- Given a polynomial function, the rational zeros will be in the form of _____ where p is a factor of the _____ (or constant) term and q is the factor of the _____ coefficient.

List all the possible rational zeros of $f(x) = 2x^3 + 2x^2 - 3x + 9$

Use the Rational Zero Theorem and Synthetic Division to Find Zeros of a Polynomial

To find all the zeros of polynomial functions,

1. Use the Rational Zero Theorem to _____ all possible rational zeros of the function.
2. Use _____ division to test a possible zero. If the remainder is 0, it is a zero. The _____ on a graph are zeros, so a graph can help you choose which possible zero to test.
3. Repeat step two using the _____ polynomial with synthetic division. If possible, continue until the depressed polynomial is a _____.
4. Find the zeros of the _____ function by factoring or the quadratic formula.

Find all rational zeros of $f(x) = x^3 - 4x^2 - 2x + 20$

The Fundamental Theorem of Algebra

- If $f(x)$ is a polynomial of degree $n > 0$, then $f(x)$ has _____ complex zero.
- A polynomial has the same number of _____ as its _____.

How many solutions does $x^4 - 5x^3 + x - 5 = 0$ have? Find all the solutions.

Given a function, find the zeros of the function. $f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$

Complex Conjugate Theorem

- If the complex number _____ is a zero, then _____ is also a zero.
- Complex zeros come in _____

Irrational Conjugate Theorem

- If _____ is a zero, then so is _____

Worksheet/ebook = 15

Algebra 2

4-Review

Take this test as you would take a test in class. When you are finished, check your work against the answers.

4-01

Simplify.

1. $(2x^2 + 3x - 4) + (-2x^2 - 5x + 7)$
2. $(5x^2 - 4) - (6x^2 + 4x + 17)$
3. $(2x^2 + 3x - 1)(x + 4)$
4. $(2x + 1)^2$

4-02

Factor.

5. $x^3 + 6x^2 + 5x$
6. $4x^3 + 2x^2 + 16x + 8$

Solve by factoring.

7. $3x^3 + 15x^2 + 18x = 0$
8. $2x^3 + 3x^2 - 8x = 12$

4-03

Divide with long division.

9. $(6x^3 + 13x^2 + 3x - 2) \div (2x^2 + 3x - 1)$
10. $(9x^3 + 6x^2 - 23x + 10) \div (3x - 2)$

Divide with synthetic division.

11. $(3x^3 + 7x^2 - 14x + 20) \div (x + 4)$
12. $(2x^4 + 3x^2 + 5x - 7) \div (x - 3)$

4-04

Use the remainder theorem to evaluate *f*(*x*) at the given *x* value.

13. $3x^3 - 2x^2 + x + 18$; x = 2
14. $x^4 - 5x^2 + 3x - 20$; x = −3

Determine whether the given binomial is a factor of *f*(*x*). Show work other than graphing.

15. $f(x) = x^3 - x^2 - 14x + 24; (x + 4)$
16. $f(x) = 6x^3 + x^2 - 5x - 2; (x - 1)$

4-05

List the possible rational zeros of the function.

17. $x^4 + 2x^2 - 4x + 16$
18. $2x^3 - 71x^2 + 40x - 8$

Find all the zeros of the function.

19. $f(x) = 6x^3 - 5x^2 - 12x - 4$
20. $f(x) = x^4 - x^3 + 2x^2 - 4x - 8$

Answers

1. $-2x + 3$
2. $-x^2 - 4x - 21$
3. $2x^3 + 11x^2 + 11x - 4$
4. $4x^2 + 4x + 1$
5. $x(x + 1)(x + 5)$
6. $2(2x + 1)(x^2 + 4)$
7. −3, −2, 0
8. −2, −3/2, 2
9. 3x + 2
10. $3x^2 + 4x - 5$
11. $3x^2 - 5x + 6 + \frac{-4}{x+4}$
12. $2x^3 + 6x^2 + 21x + 68 + \frac{197}{x-3}$
13. 36
14. 7
15. Yes
16. Yes
17. ±1, ±2, ±4, ±8, ±16
18. ±1/2, ±1, ±2, ±4, ±8
19. −2/3, −1/2, 2
20. −1, 2, ±2i